

*Full Length Research Paper*

# Level of student's creative thinking in classroom mathematics

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It is reasonable to assume that people are creative, but the degree of creativity is different. The idea of the level of student's creative thinking has been expressed by experts, such as Gotoh (2004), and Krulik and Rudnick (1999). The perspective of the mathematics creative thinking refers to a combination of logical and divergent thinking which is based on intuition but has a conscious aim. The divergent thinking is focused on flexibility, fluency, and novelty in the mathematical problem solving and problem posing (Silver, 1997). Students have various backgrounds and different abilities. They possess different potential in thinking pattern, imagination, fantasy and performance. Therefore, students have a different level of creative thinking. This research used qualitative approach which aims to describe the characteristic of the level of student's creative thinking in mathematics. The task-based interview was conducted to collect data from the 8<sup>th</sup> grade students of junior secondary school. Snowball method was used to determine subject research. Finally, there were nine students from junior secondary school of "SMP Negeri 6 Sidoarjo" and one student from "SMP Al Hikmah" Surabaya. The result of this research pointed out the five levels of creative thinking that are of level 0 to level 4 which has a different characteristic. This difference is based on fluency, flexibility, and novelty in mathematical problem solving and problem posing.

**Key words:** Student's creative thinking, problem posing, flexibility, fluency, novelty

## INTRODUCTION

It is reasonable to assume that people are creative, but the degree of creativity is different (Solso, 1995). This fact was shown by someone who created technology or knowledge at the disposal of others using it. This observation points to the existence of different levels or degrees of creativity or creative thinking for different people.

The idea of the level of students' creative thinking has been expressed by experts. De Bono (Barak and Doppelt, 2000) defined four achievement levels of creative thinking skills development. These are awareness of thinking, observation of thinking, thinking strategy, and reflection on thinking. These levels are too general and not easily recognized on the mathematical perspective. Gotoh (2004) described three stages of development of mathematical thinking in problem solving. They are the empirical (informal) activity (stage 1), the algorithmic (formal) activity (stage 2), and the constructive (creative) activity (stage 3). In similar terms, Eryvnyck (Sriraman, 2005) presented three stages of

mathematical creativity namely: preliminary technical stages (stage 0), algorithmic activity (stage 2), and creative (conceptual, constructive) activity (stages 3). The preliminary technical stage consists of some kind of technical or practical application of mathematical rules and procedures, without any awareness of the theoretical foundation from the user. Algorithmic activity consists primarily of performing mathematical techniques, such as explicitly applying an algorithm repeatedly. Creative activity consists of non-algorithmic decision making. Krulik and Rudnick (1999) also described the levels of thinking as recall, basic, critical, and creative thinking. Recall includes those skills that are almost automatic or reflexive. Basic includes the understanding and recognition of mathematical concepts like addition, subtraction, etc, as well as their application in problems. Critical thinking is thinking that examines, relates, and evaluates all aspects of a situation or problem. Creative thinking is thinking that is original and reflective and that produces a complex product. Those levels are also not

easily predictable or identified in the learning process.

The perspective on mathematics creative thinking refers to a combination of logical and divergent thinking which is based on intuition but has a conscious aim, as pointed out by Pehkonen (1997). Divergent thinking is focused on flexibility, fluency, and novelty (Krutetskii, 1976; Haylock, 1997; Silver, 1997) in mathematical problem solving and problem posing.

Silver (1997) pointed out an indicator to identify students' creative thinking (fluency, flexibility, and novelty) by using problem solving and problem posing. Three components respectively assessed different parts and were independent of each other. Students have various backgrounds and different abilities. They possess different potentials in thinking pattern, imagination, fantasy and performance. Therefore, students have different levels of creative thinking. A student may either achieve three components, two components, or only one component.

The development of a set of levels for students' creative thinking actually has been done, but only in terms of problem posing in mathematics (Siswono, 2004). The descriptions of these levels are shown below:

**Level 5:** Result of student's task satisfied all criterion of creativity product. Student can synthesize ideas, generate new ideas from mathematical concepts and real life experience, and apply the ideas to construct some problems also revised when they find a hindrance.

**Level 4:** Result of student's task satisfied all criterion of creativity product. Student can synthesize ideas, generate new ideas from mathematical concepts and little real life experience, and apply the ideas to construct some problems also revised when they find a hindrance.

**Level 3:** Result of student's task satisfied all criterion of creativity product. Student can synthesize ideas, generate new ideas only from mathematical concepts, and apply the ideas to construct some problems also revised when they meet a hindrance.

**Level 2:** Result of student's task satisfied just one or two criterion of creativity product. Student can synthesize ideas from mathematical concepts or real life experience, and generate new ideas only from mathematical concepts or real life experience. He/She has not applied all ideas to construct some problems, but he/she can revise a problem when they find a hindrance.

**Level 1:** Result of student's task satisfied just one or two criterion of creativity product. Student can not synthesize ideas from mathematical concepts or real life experience, but can generate new ideas only from mathematical concepts or real life experience. He/She has not applied all ideas to construct some problems also revised when they find a hindrance.

**Level 0:** Result of student's task did not satisfy all criterion of creativity product. Student can not synthesize ideas from mathematical concepts or real life experience, and can not generate new ideas. They just recall their ideas.

These levels are verified to students at two junior high school, at grade 7 (SMP Negeri 4 and SMP Negeri 26 Surabaya). All students are given the problem posing tasks which the information is based on a picture/diagram or word problem (verbal) situation. Some students were chosen to be interviewed in depth. The results pointed out that students were placed at all level and it was not impacted by students achievement's level or sexuality. These indicated that the problem posing task could be appropriated to classify the level of students creative thinking. Siswono and Novitasari (2006) pointed out that problem posing activities by "what's another way strategy" could improve students abilities in creative thinking.

Based on these facts, The levels of student's creative thinking were revised. It was not only a mathematical problem posing, but also emphasized a mathematical problem solving. This level is a hypothetical theory which consists of 3 components; they are flexibility, fluency, and novelty in mathematical problem solving and problem posing. They are called the draft of levels of creative thinking (LCT) that consisted of 5 levels. Those levels that emphasized divergent thinking and also where the highest ordered were novelty, then flexibility and the least aspect was fluency. Novelty was placed at the highest position because it was the main characteristic to assess the product of creative thinking. Flexibility was placed as the next important position because it referred to the production of some ideas which were used to solve a task. Fluency was indicated when the student fluently produced different ideas which were appropriate to the question task. The draft was verified by using student's data (Siswono and Budayasa, 2006) as the initial research. It was found from the results of previous research that students with characteristics of LCT were in levels 4, 1 and 0, and were not in levels 2 and 3. Even though not all levels were evident among the students, it was enough to verify the theory. In this present research, the draft was revised so that novelty and flexibility become important components, but no one was higher than the others as in the previous theory. This set of levels is called the revised draft of creative thinking levels. It is a hypothesis which will be verified in the mathematics classroom.

The focus of this research is to describe the characteristics of students' creative thinking levels. Creative thinking is the mental process which someone uses to come up with the "new" ideas as fluency and flexibility. "Idea" means a thought in solving and posing a problem. Mathematics problem posing is a task which asks students to pose or construct a mathematical

problem based on the given information, and then solve the problem. Fluency in problem solving refers to a student's ability to obtain many solutions to a problem. Fluency in problem posing refers to certain kind of problems with the correct solutions which are posed by students. Some solutions are of the same kind as the others when they have the same pattern, such as, the type of shape is the same but the size is different. Some solutions are different when they have a different pattern or are not usual for student grades, such as, students can construct a combination of other shapes. In problem posing, some problems are of the same kind as others when they use the same concept, but with the different attributes, the problem will be common and recognized by many students. Two problems are different when they have different contexts and concepts or are unfamiliar to students.

Flexibility in problem solving refers to a student's ability to solve a problem using many different methods or ways. Flexibility in problem posing also refers to a student's ability to pose or construct problem with divergent solutions.

Novelty in problem solving refers to the student's ability to solve a problem with many different solutions and correct answers, especially to find an original solution which is not common for that student's grades or their knowledge level. Novelty in problem posing refers to a student's ability to pose or construct a problem different from others.

## METHOD

The research approach is qualitative research which aims to identify the characteristics of the level of students' creative thinking in mathematics. Data was collected through the task-based interview to the 8<sup>th</sup> grade students of junior secondary school and analyzed by using the constant comparison method. The method for determining a sample subject used the snowball method. Finally, the subjects were nine students from junior secondary school of "SMP Negeri 6 Sidoarjo" and one student from "SMP Al Hikmah" Surabaya. They were in the higher ability group of mathematics at their school and they had good communication skills. The task was an open-ended task which was divergent in its solution (answer) and methods (ways). Triangulation was conducted by giving students another equivalent task and interviewing them again deeply. The students' work was analyzed by identifying the correctness of the answers, then checking for aspects of creative thinking (fluency, flexibility, and novelty) in problem posing and solving. Students' level was estimated by applying a qualitative analysis method to determine the level of a student's creative thinking.

## RESULTS

The result of this research describes the five hierarchical levels of students' creative thinking in mathematics. These levels are shown in Table 1. A characteristic of each level is established by a constant comparison of two students' characteristics on the same level. For example,

student PE and ARP on level 4 have the same characteristics as shown in Table 1. However, they have different characteristics as well, such as:

(1) PE's method to construct another polygon with the area which is the same as the area of a rectangle 12 by 8 cm is:

(a) Determine and draw a kind of polygon, then determine its appropriate dimension.

(b) Cut a model of a rectangle (paper) as a simple shape and change it into other kinds of polygon. (See part of interview).

Although ARP's method at first is the same as PE's method, another ARP's method is different. First, she gives a dash line to a rectangle model, and then by her imagination, set it up into another shape that she wants.

**Interviewer:** Okay. Based on a rectangle 12 by 8 cm, you can make another polygon with the same area that is trapezoid. How do you make it?

**PE:** I drew its shape....then by trying out. Its height was 8 then...given name  $A, B, C, D...$   $AB$  which is equal with 10 cm and  $CD$  is 14 cm...I checked it using trapezoid formula. Like this  $24 \times 4$  equal with 96 square centimeter.

**Interviewer:** Now, how is the other way to make this trapezoid?

**PE:** By cutting....

**Interviewer:** Well, can you demonstrate it how you made it?

**PE:** (She took a paper and wrote down a dimension as a dimension of given rectangle and cut it with scissors to make a trapezoid. PE took a scissors herself).

(2) PE can make a problem related to real life. She makes the form of polygon as the traffic sign, or windows. According to PE, an important thing is a related concept, question, and methods or way for solving. ARP's problem is not related by real life, because she thinks a sentence is too long and not easy to solve. (Figure 1)

## DISCUSSION

The characteristics of the levels of creative thinking as shown on Table 1 contained a different primary aspect for each level. The difference is in the creative thinking aspects that consisted of flexibility, novelty, and fluency in mathematical problem solving and problem posing. However, there are similar common characteristics among those levels. At level 4 and 3, students tend to

**Table 1.** Characteristics of a student's creative thinking level.

Level	Characteristic of creative thinking level
Level 4 (Very Creative)	Student is able to solve a problem with more than one solution and can represent another way to solve it. One solution fulfills originality (novelty). He/she also can pose novel problems. One problem has divergent solutions and divergent methods to solve it. Some constructed problems fulfill novelty, fluency and flexibility. He/she tends to say that constructing a problem is more difficult than solving a problem, because he/she must have a certain way to make its solution. He/she tends to say that finding the solution method is more difficult than searching for other answers or solutions.
Level 3 (Creative)	Student is able to solve a problem with more than one solution, but he/she cannot represent another way to solve it. One solution fulfills originality (novelty). An alternative characteristic, he/she can represent another way to solve a problem, but he/she cannot make a novelty solution. On the other hand, he/she can also pose novelty problems. One problem has divergent solution, but no divergent method to solve it. Or, he/she can make divergent method for one constructed problem but no such problem fulfills novelty. He/she tends to say that constructing a problem is more difficult than solving a problem, because he/she must have a certain way to make his/her solution. He/she tends to say that finding the solution method is more difficult than searching other answer or solution.
Level 2 (Quite Creative)	Student is able to solve a problem with one original solution however it does not fulfill fluency or not flexibility. Or, he/she can represent another way to solve a problem; however, it is not novelty or not fluency. Another characteristic, he (or she) also can pose novelty problems without fluency and flexibility. Or, some constructed problems fulfill flexibility without novelty and fluency. He/she tends to say that constructing a problem is more difficult than solving a problem, because he/she is not familiar with a task and difficult to estimate numbers, formula or solutions. He/she tends to understand that the different method or strategies to solve a problem as another formula with different representation.
Level 1 (Almost Not Creative)	Student is able to solve a problem with more than one solution but cannot represent another way to solve it. The solution does not fulfill originality (novelty). He/she also can pose some problems. However the problem has no divergent solution and method. The constructed problems just fulfill fluency without novelty and flexibility. He/she tends to say that constructing a problem is quite difficult than solving a problem, because it depends on the complexity of problem. He/she tends to understand that the different method or strategies to solve a problem is another form of formula, though those are same. Problems tend mathematically without connecting to real life.
Level 0 (Not Creative)	Student cannot solve a problem with more than one solution and cannot represent another way to solve it. Solutions do not fulfill originality (novelty), fluency and flexibility. He/she also cannot pose the novelty and flexibility problems. All constructed problems do not fulfill novelty, fluency and flexibility. His/her mistakes are caused by weakness of understanding the related concepts. He/she tends to say that constructing a problem is easier than solving a problem, because he/she know its solution. He/she tends to understand that the different method or strategies to solve a problem as another formula with different representation.

say that constructing a problem is more difficult than answering a problem directly and finding another way or method is more difficult than finding another answer or solutions. At level 2, 1, and 0, students tend to understand other methods to solve a problem as another formula with different representation. For example, formula for the perimeter of rectangle;  $2(p + l)$  is different from  $2p + 2l$ . Actually, those formulas are the same. Those grades of level pointed out the gradation of students' ability. Where the higher ability level is, the higher creative thinking level becomes.

Students at level 0 tend to consider that posing a

problem is easier than solving a problem. Meanwhile, students at level 1 tend to state that to construct a problem is not difficult, but it is not easier than to solve a problem. Students at levels 2 to 4 tend to state that constructing a problem is more difficult than solving a problem. This difficulty is caused by the complexities to estimate the given information, make an appropriate sentence, and construct its solutions.

This result is similar to the findings of Siswono (1999). Students at the lowest group tend to state that to pose a problem is easier because they can make a problem suitable to their abilities. The higher group tends to



**Figure 1.** PE takes a scissors herself.

explain that to pose a problem is more difficult than to solve a problem. The reason is that answering a problem didn't require them to think about the form of the problem and because it is a familiar task, they usually find a solution. This tendency becomes the feature of different levels of creative thinking. However, it does not guarantee that if a student is in a higher group and states that constructing a problem is more difficult than solving it, and then she/he is classified as a student at higher level of creative thinking. It depends on the student's ability to fulfill some components of the characteristics of mathematics creative thinking.

Some students who were subjects of this research were at the same level but they did not possess identical characteristics, such as at level 3. Student AF fulfilled fluency and flexibility, but he didn't fulfill novelty in posing and solving problems. Meanwhile, student RF fulfilled fluency and novelty but didn't fulfill flexibility in solving and posing problems. Both are at level 3 because novelty and flexibility have been classified as having similar degree. It means that both aspects are important or are fundamental characteristics of mathematics creative thinking.

At level 4, students fulfilled the three aspects of creative thinking or fulfilled flexibility and novelty, but they did not satisfy fluency in problem solving and problem posing. Both students as research subjects at this level have fulfilled the three aspects. It is related to the form of a task as a research instrument. A task may be designed to be divergent in solutions and ways or methods. Students will satisfy flexibility and novelty when the tasks have only divergent solutions.

## Conclusion

This research has described the characteristics of the student's creative thinking levels. The difference of the levels is based on fluency, flexibility, and novelty in mathematical problem solving and problem posing. Students at level 4 fulfilled three components of creative thinking indicators; level 3 fulfilled two components,

flexibility and fluency, or novelty and fluency. Students at level 2 only satisfied one aspect that is flexibility or novelty, and at level 1 only satisfied a fluency aspect. Students at level 0 did not fulfill all components. These levels are easier to apply in the mathematics classroom because teachers can examine the product of the task if their objective is to develop students' creative thinking in mathematics.

This research is one of the approaches to assess, identify or classify students' creative thinking in mathematics. The study of creative thinking or creativity has many limitations because creative thinking or creativity is a multi-faceted phenomenon. It arises from many definitions, criteria, or concepts. However, it is quite possible to focus on certain aspects, as pointed by Isaksen (2003) that "It is quite possible that various researchers and writers emphasize certain facets of creativity in their definitions because of the focus of their work". Finally, I hope this research will stimulate others to continue the research, verify, modify, or apply it.

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